



# The Industrial Use of Formal Methods: Experiences of an Optimist

Prof. Jonathan P. Bowen

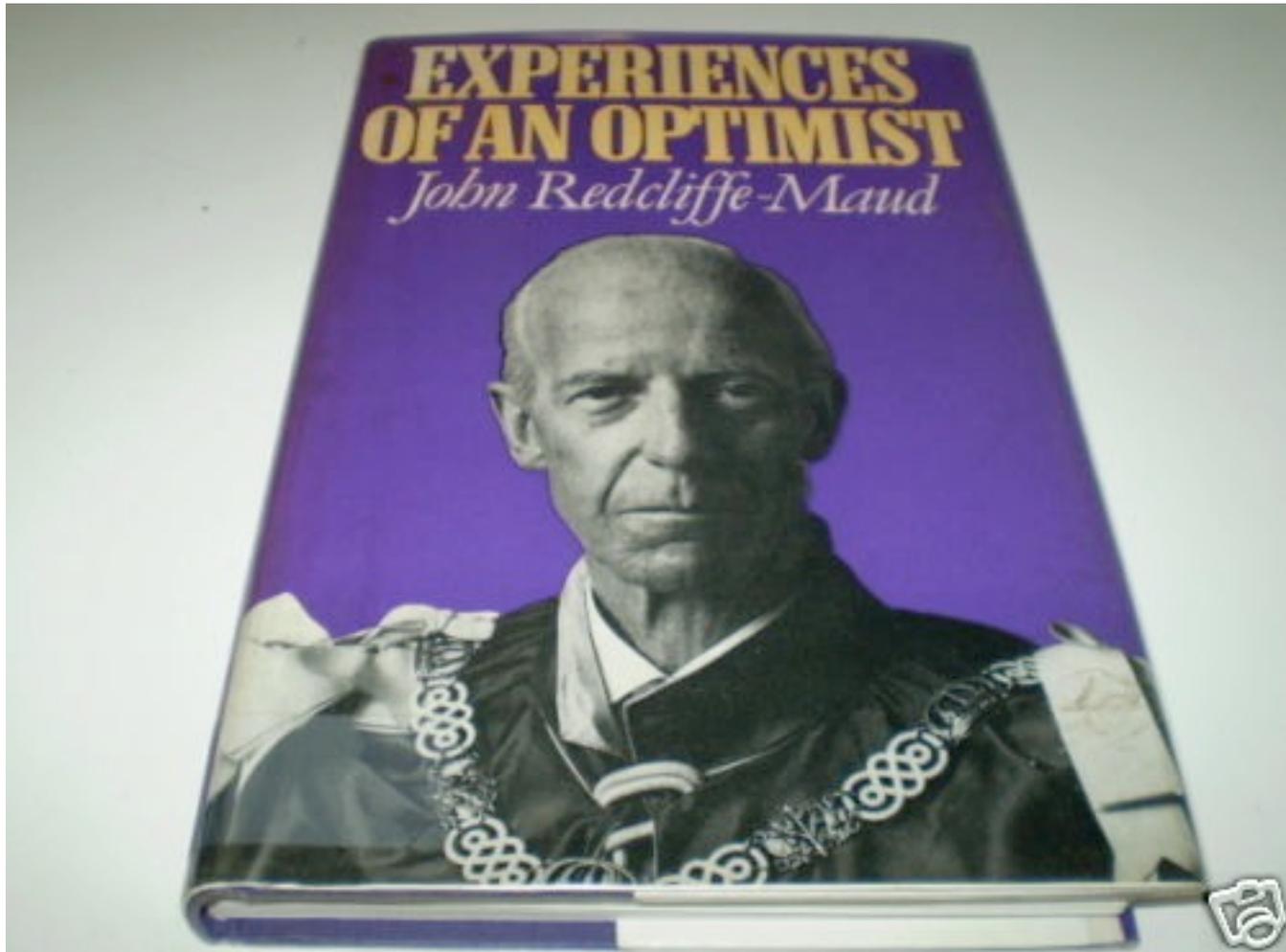
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# Experiences of an Optimist



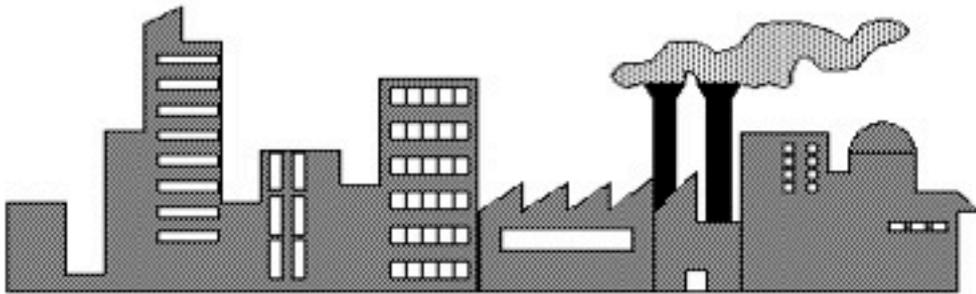
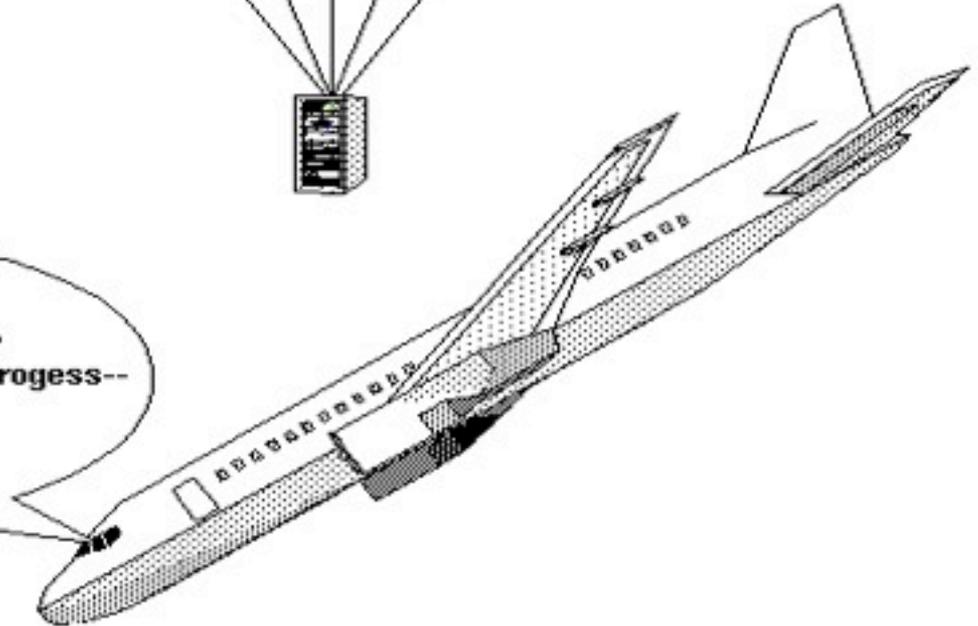
[http://en.wikipedia.org/wiki/John\\_Redcliffe-Maud](http://en.wikipedia.org/wiki/John_Redcliffe-Maud)



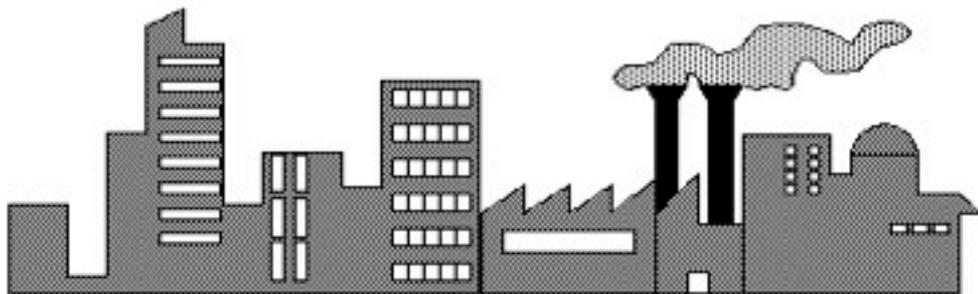
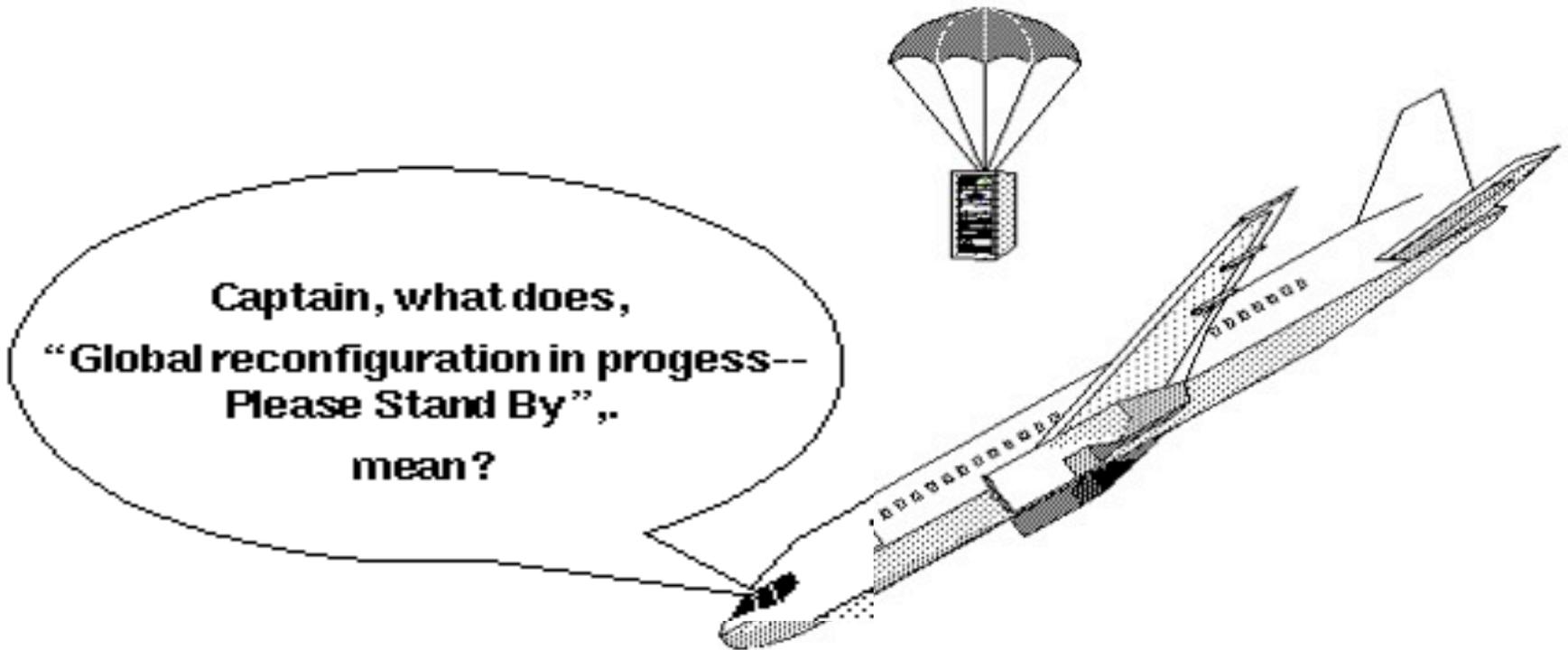
# Background: Safety and reliability



Captain, what does,  
"Global reconfiguration in progress--  
Please Stand By",  
mean?



# Background: Safety and reliability



# Airbus A380 simulator



Emirates Aviation College  
Dubai, 3 February 2011



# Theory and Practice

*“It has long been my personal view that the **separation of practical and theoretical work is artificial and injurious**. Much of the **practical work** done in computing, both in software and in hardware design, is **unsound and clumsy** because the people who do it have not any clear understanding of the fundamental design principles of their work. Most of the abstract mathematical and **theoretical work** is **sterile** because it has no point of contact with real computing.”*

— [Christopher Strachey](#) (1916–1975)



# Formal Methods



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- Term established by late 1970s
  - Next stage from structured design
  - Mathematical basis



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  - Validation (correct specification)
  - Verification (correct implementation wrt spec)



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- Formal specification and (optionally) proof:
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- But engineers *calculate* rather than prove
- Please contribute to the *Formal Methods Wiki*:
  - <http://formalmethods.wikia.com>



# Z notation



# Z notation

- Formal specification – predicate logic, set theory, and schema boxes
  - Courses (academia & industry)
  - Textbooks (reasonable choice)
  - Tools (type-checkers, provers, ...)



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- Google group – [comp.specification.z](https://groups.google.com/group/comp.specification.z)



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- Z User Group (meetings) & Z standard



# Z Standard



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- ISO/IEC 13568
  - Long process (1990s)
  - Inconsistencies found!



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- Final Committee Draft
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# Z Standard



- ISO/IEC 13568
  - Long process (1990s)
  - Inconsistencies found!
- Final Committee Draft
  - accepted in 2001
- Useful for tools and industrial application



# Levels of Complexity – Abstraction



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- 25 lines of informal requirements



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- 250 lines of specification (e.g., Z)



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# Levels of Complexity – Abstraction

- 25 lines of informal requirements
- 250 lines of specification (e.g., Z)
- 2,500 lines of design description
- 25,000 lines of high-level program code
- 250,000 machine instructions of object code
- 2,500,000 CMOS transistors in hardware!



# Technology transfer problems



The only thing harder to sell than formal methods.



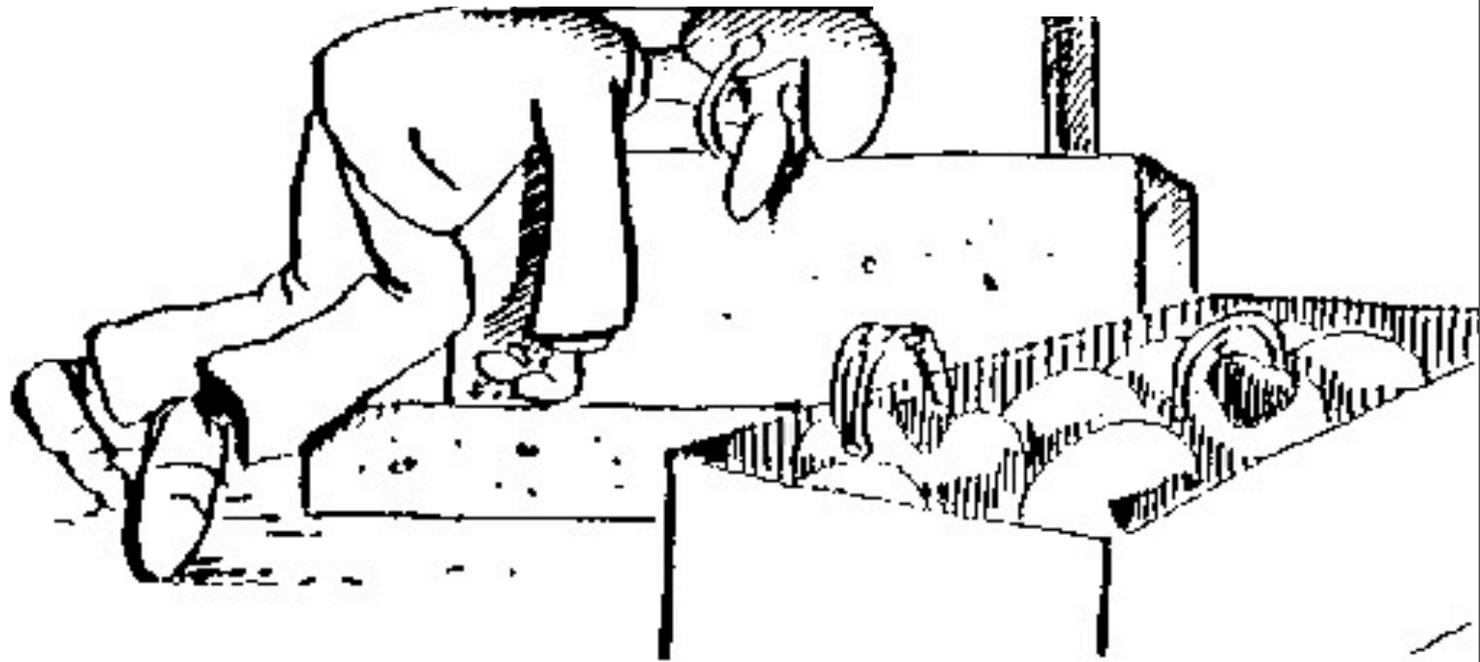
# Choosing a formal method – difficult



Choosing a formal method can be a fearful thing.



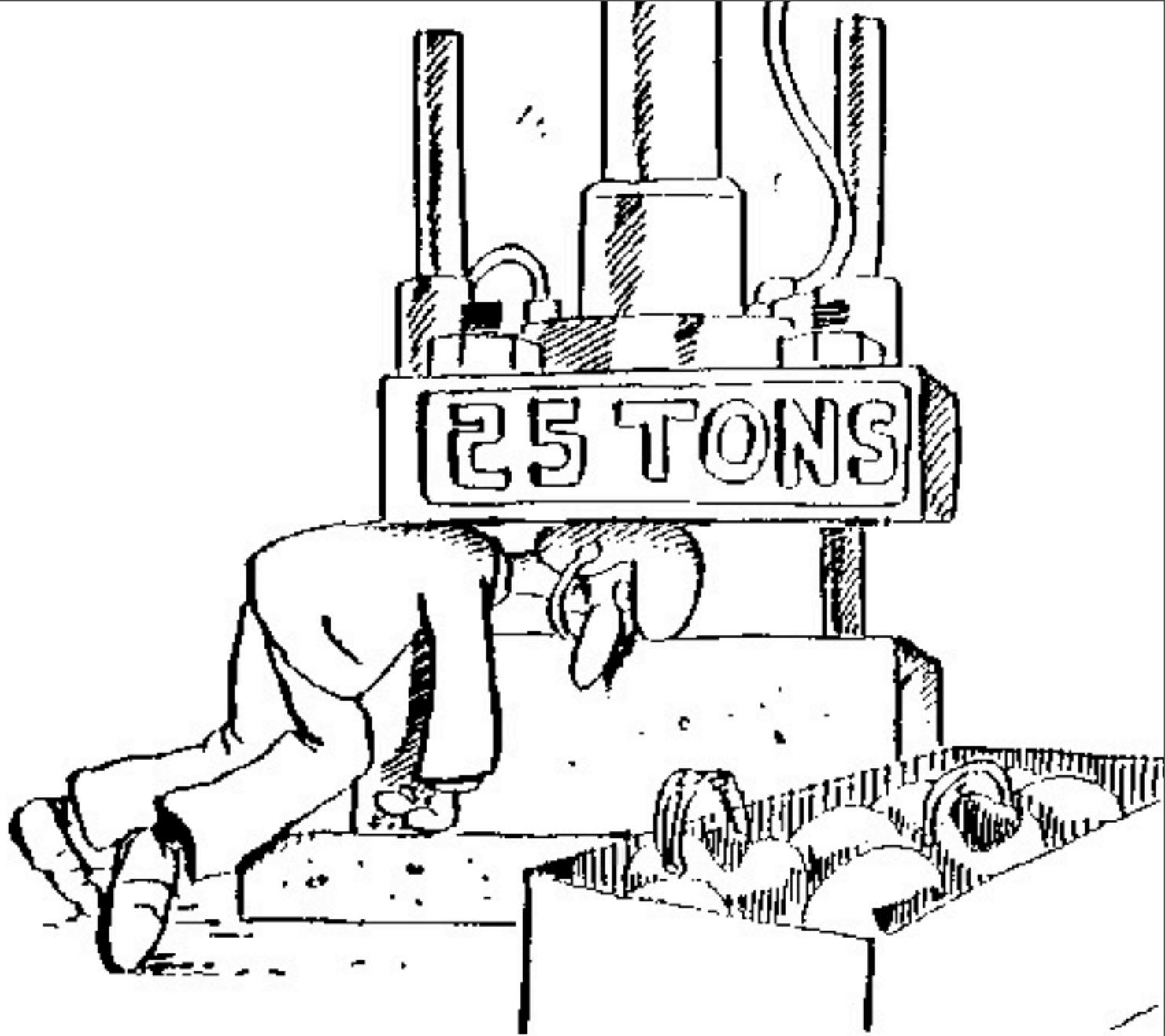
# Tools – difficult to use



What it is like to use a mechanical theorem prover.



# Tools – difficult to use



What it is like to use a mechanical theorem prover.



# Applications of Formal Methods



# Applications of Formal Methods

Examples:



# Applications of Formal Methods

Examples:

- Tektronix (Z)
- STV algorithm (VDM)
- IBM CICS (Z/B)



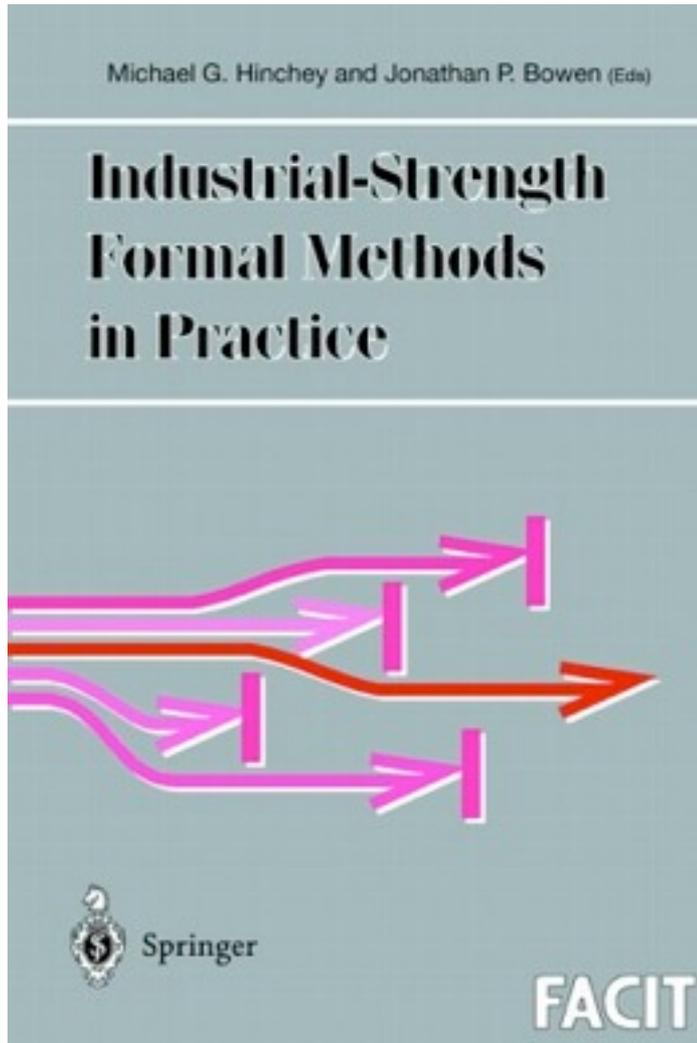
# Applications of Formal Methods

Examples:

- Tektronix (Z)
- STV algorithm (VDM)
- IBM CICS (Z/B)
- AAMP5  $\mu$ processor (PVS)
- GEC Alsthom (B)
- A300/340 (Z)

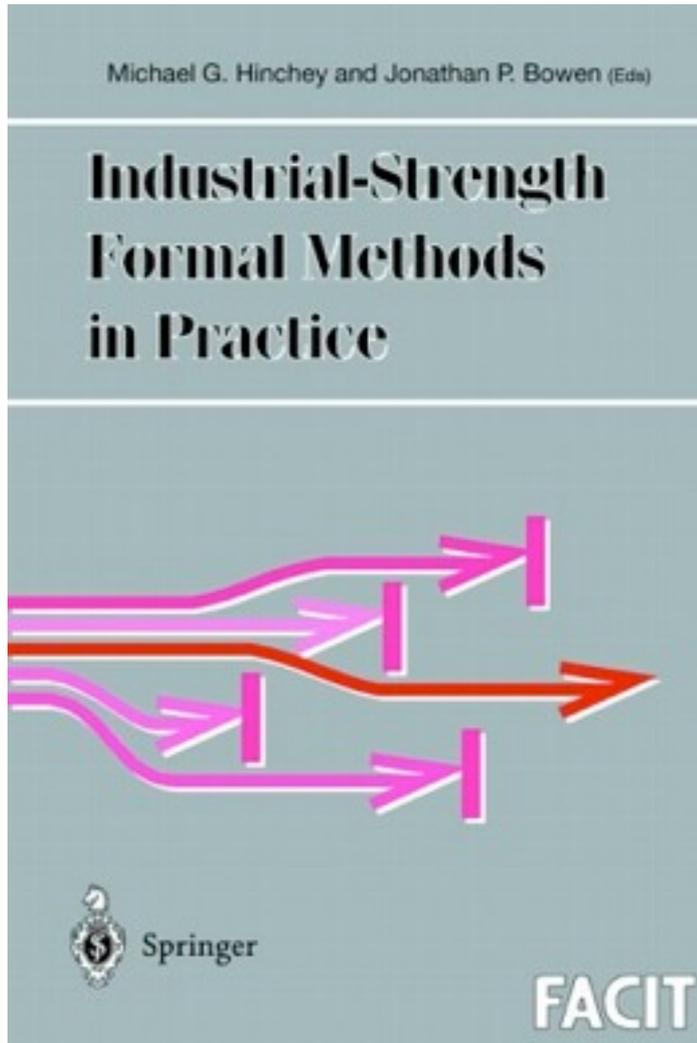


# Industrial-Strength Formal Methods in Practice

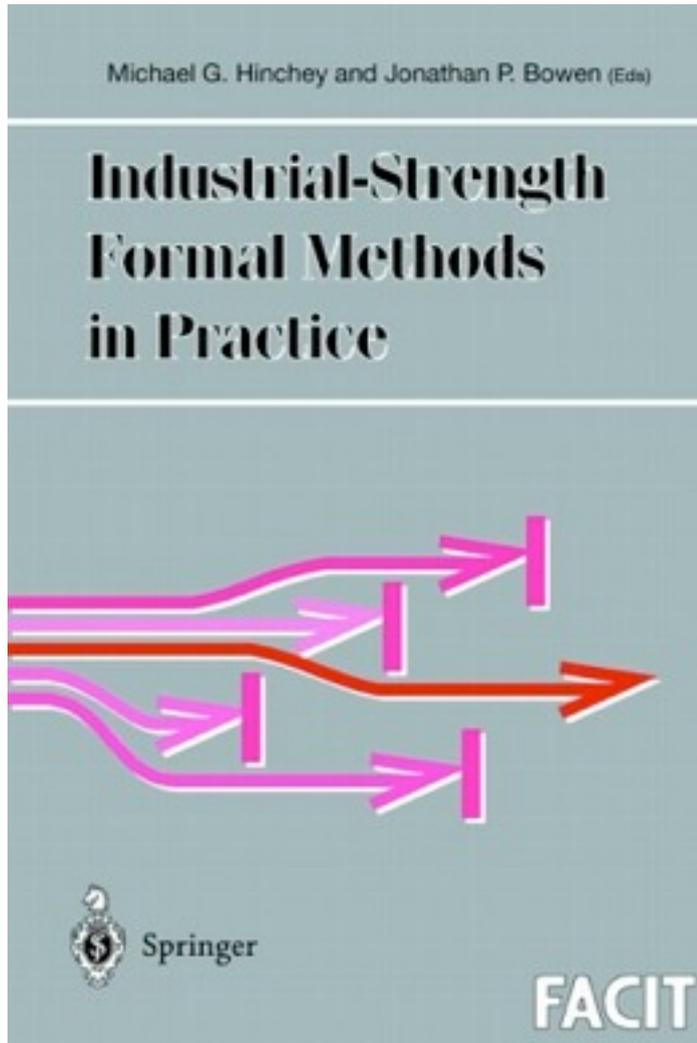


# Industrial-Strength Formal Methods in Practice

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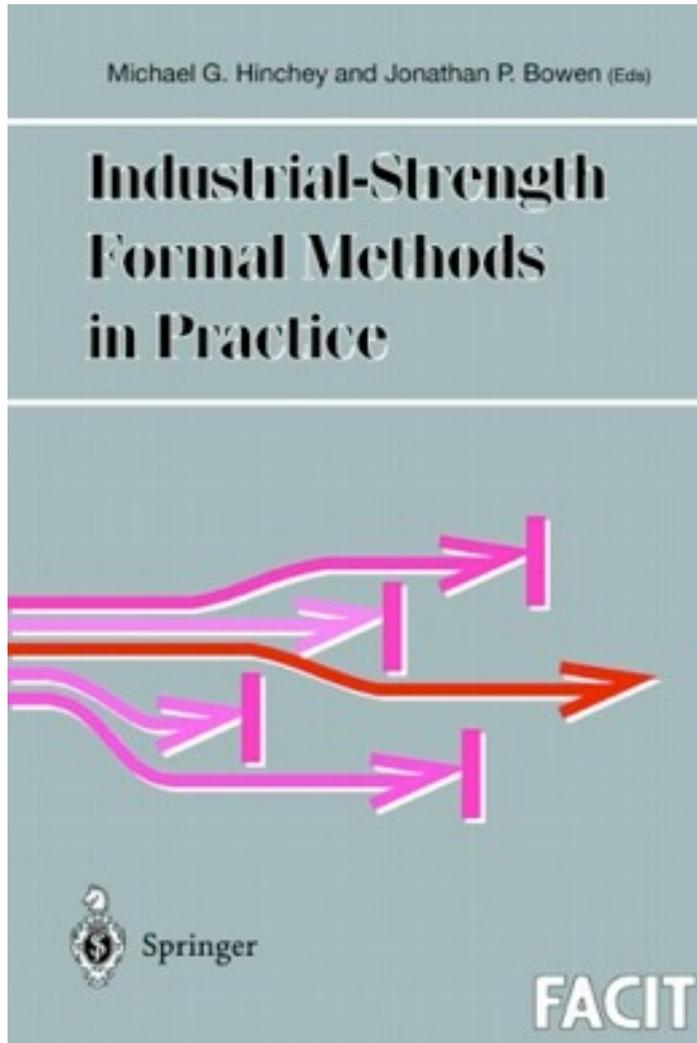


## Examples:

- Motorola CAP DSP (ACL2)
- Radiation Therapy Machine (Z)



# Industrial-Strength Formal Methods in Practice

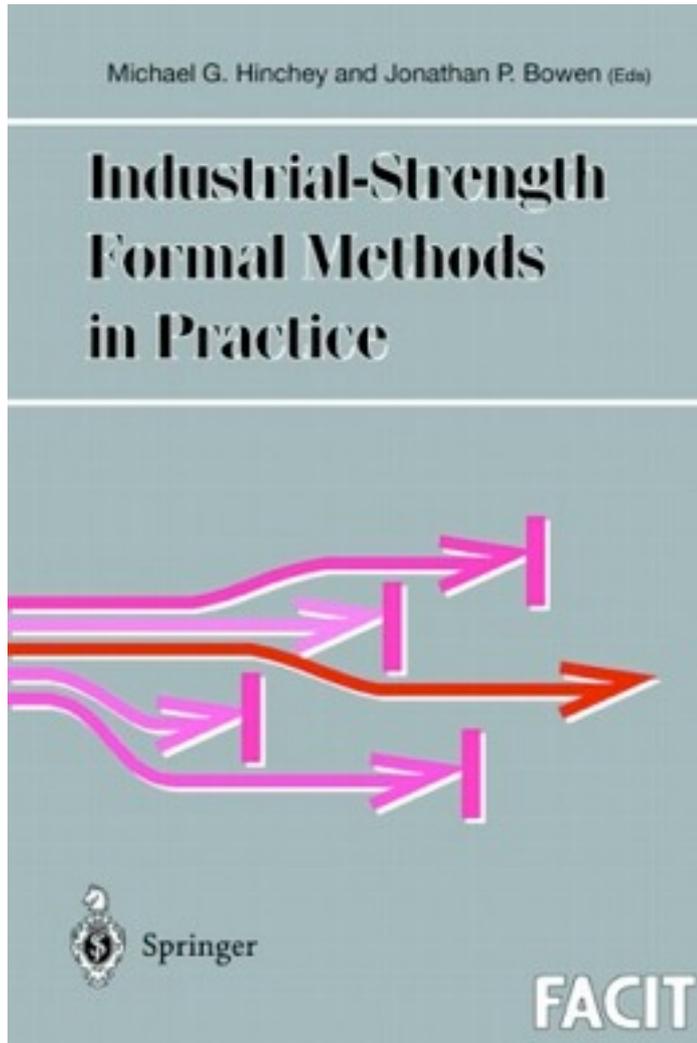


## Examples:

- Motorola CAP DSP (ACL2)
- Radiation Therapy Machine (Z)
- ATC system (VDM)
- Railways (Prover Technology)



# Industrial-Strength Formal Methods in Practice



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- Railways (Prover Technology)

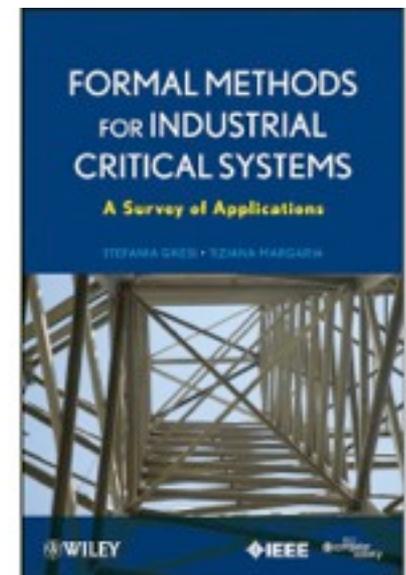
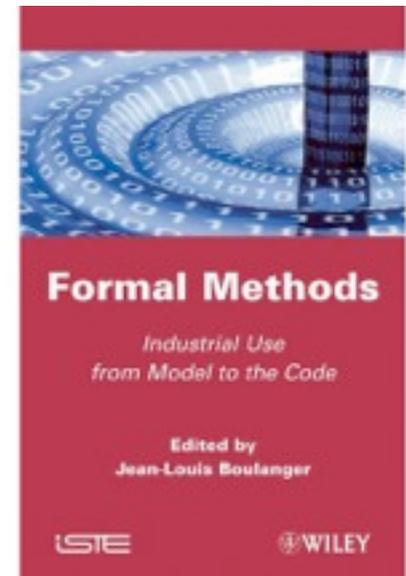
And more recently:

Microsoft



# Recent books

- Boulanger, J.-L., ed. 2012.  
*Formal Methods: Industrial Use from Model to the Code.*  
ISTE, Wiley.  
ISBN 978-1848213623.
- Gnesi, S. and Margaria, T. 2012.  
*Formal Methods for Industrial Critical Systems: A Survey of Applications.*  
IEEE Computer Society Press, Wiley.  
ISBN 978-0470876183.



**NATS**

# **National Air Traffic Services**

NATS

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- Provides air traffic control from its centres at **Swanwick**, Hampshire and **Prestwick**, Ayrshire.
- Also provides air traffic control services at 15 of the UK's major airports including **Heathrow**, Gatwick, Stansted, Birmingham, Manchester, Edinburgh, and Glasgow, together with air traffic services at Gibraltar Airport.

# **NATS** National Air Traffic Services, UK



Swanwick  
southern England

[www.nats.co.uk](http://www.nats.co.uk)

# Flight strips on paper



	RVA 83A	SPEEDBIRD <b>BAW2</b> 4235	SLS+	
03E 24		CONC/W T560	KJFK	F600 EGLL

# Flight strips on paper



03E 24	RVA 83A	SPEEDBIRD <b>BAW2</b> 4235	SLS+	F600
		<b>CONC</b> T560	KJFK	EGLL

Last flight of Concorde







# NATS

## National Air Traffic Services

- Advertisement & leaflet at Heathrow Airport →
- Air Traffic Management (ATM)
- Single European Sky ATM Research (SESAR)
- SESAR Joint Undertaking
- [www.sesarju.eu](http://www.sesarju.eu)
- SESAR project (2004–20)

go to [heathrowairthought.com](http://heathrowairthought.com) Heathrow Making every journey better

worldcityline  
ferrovial  
APCOA  
BRITISH AIRWAYS  
VIRGIN Atlantic  
NATS  
AOC  
onl  
British Sky  
JCCOxix Airport  
Hertz  
Miles

To lighten our environmental footprint we're standing together.

1

HeathrowAirthought  
Together towards sustainability

# NATS

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go to [heathrowairthought.com](http://heathrowairthought.com) Heathrow  Making every journey better

worldstyle ferrovial APCOA

**NATS**

JCCOx Airport Hertz

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1

HeathrowAirthought  
Together towards sustainability



[www.altran-praxis.com](http://www.altran-praxis.com)

Open-DO

# Formal Methods in Air Traffic Control

Slides by Neil White

[www.slideshare.net/AdaCore/white-open-do](http://www.slideshare.net/AdaCore/white-open-do)  
[www.youtube.com/watch?v=IQMWVqQfm5A](http://www.youtube.com/watch?v=IQMWVqQfm5A)

# Agenda

- A quick introduction
  - What is iFACTS?
- Formal methods for Specification
  - Z, State machines.
- Formal methods for Implementation
  - Implementation: SPARK.
- Formal methods for Test
  - Verification: more Z, Mathematica.

# Context

- NATS, the UK's leading air traffic services provider, has pioneered research and development of advanced air traffic control tools for several years from its simulator and research centre. The iFACTS project will deliver a subset of these tools onto the system at the company's main en-route Control Centre at Swanwick.
- Further early information is available at: [www.computerweekly.com/Articles/2007/03/07/222258/Nats-claims-the-biggest-air-traffic-control-innovation-since.htm](http://www.computerweekly.com/Articles/2007/03/07/222258/Nats-claims-the-biggest-air-traffic-control-innovation-since.htm)



# ATC team



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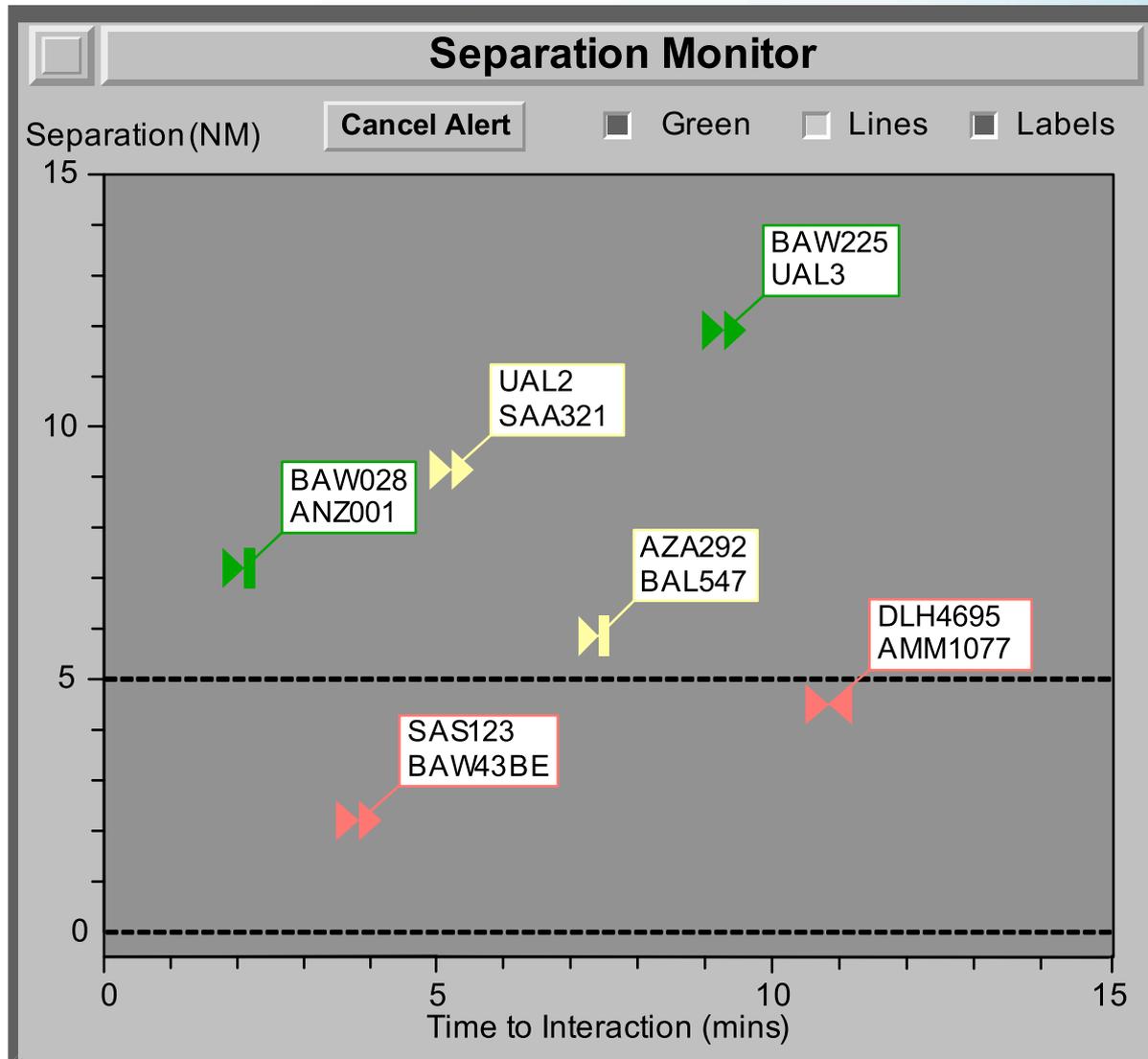
# Why iFACTS?

- iFACTS – Interim Future Area Control Tools Support – to further improve safety and provide Controllers with a set of advanced support tools, which enables them to increase the amount of traffic they can comfortably handle. In trials, the system delivered significant capacity increases.

# What is iFACTS?

- iFACTS provides tools to support the controllers
  - Electronic flight strips replace the paper flight strips.
  - Trajectory tools – including prediction, deviation alerts, and conflict detection – are added.
- iFACTS is not an Air Traffic control system
  - Integrated with, but sits alongside, the existing system.

# Medium Term Conflict Detection:



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# The complete iFACTS specification

- The functional specification
  - Z
- The algorithm specification
  - Maths
- The HMI specification
  - State tables
- The rest of the specification!
  - English

# The Z specification

Every flight is associated with an aircraft type if its aircraft type name matches.

Every flight is associated with a performance model. If there is no model corresponding to the aircraft type, then this is a default model. If there is a filed speed up to *maxPistonSpeed*, then this is *unknownPiston*, if there is a filed speed above this but no greater than *maxTurbopropSpeed*, it is *unknownTurboprop*, otherwise it is *unknownJet*.

The set *fwpNASDeletedFlights* is those flights that have been NAS deleted. It is used by Recognised Flights.

```

FWPFlightsDerivedAssociations
FWPFlightsAssociations
fwpFlightAircraftType : FLIGHT → AIRCRAFTTYPE
fwpFlightPerformanceModel : FLIGHT → PERFORMANCEMODEL
fwpNASDeletedFlights : F FLIGHT
-----
fwpFlightAircraftType = {f : fwpFGFlights ; a : aircraftTypes
  | (fwpFlightState f).aircraftTypeName = a • f → a}
fwpFlightPerformanceModel =
  {f : (fwpFGFlights \ fwpBlockers), model : PERFORMANCEMODEL |
    (let speed = (fwpFlightState f).filedSpeed •
      model = if the speed ≤ maxPistonSpeed then unknownPiston
              else if the speed ≤ maxTurbopropSpeed then unknownTurboprop
              else unknownJet)}
  ⊗ (fwpFlightAircraftType ; typePerformanceModel)
fwpNASDeletedFlights = {f : fwpFGFlights | (fwpFlightState f).nasDeleted = True}

```

# Z training

- Z reader training
  - 3 day course; fluency then comes after 1 week on the job.
  - We have trained 75 people to read Z.
  - Engineers, domain experts, ATCOs.
- Z writer training
  - 3 day course, fluency then comes after 3 months on the job.
  - We have trained 11 people to write Z.
  - All engineers.

# Z tools

- Z written in Microsoft Word
  - To get acceptance, you need to work with what people know.
  - Supported by Word Add-ins.
    - A Z character set.
    - A simple interface to the fuzz type checker.
    - A graphical representation tool.

# Z tools

- Advantages
  - Easy to develop commentary and Z together.
  - Hyper linking of fuzz errors back to source.
  - Cross-referencing of Z names in final document.
- Disadvantages
  - All the problems of large word documents.
  - Tools can be slow on 1000 page documents.
  - Merging branches is painful.

# The state machine specification

	Button 1	Checkbox 1
State 1	State 2	N/A
State 2	State 1	State 3
State 3	State 1	State 2

## Transition Actions

State 1 -> State 2 : De-select Checkbox 1

# State machine training & tools

- Training
  - So trivial that we don't train!
  - People “just get it”.
- Tools
  - Err .... None.

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# The SPARK Implementation

- SPARK Ada
  - An annotated subset of Ada.
- 150 KSLOC (Logical)
- RTE (Run-Time Exception) Proof
  - Formal partial correctness proof against specification not considered cost-effective.

# Code

```
function Segment_Group_FL_Occupancy
(Segs_Group : PIO_Data.Segment_Group_Array_T;
Quantity    : PIO_Data.Trajectory_Index_T)
return Altitudes.Level_Range
is
  The_Range : Altitudes.Level_Range;
  Temp_Range : Altitudes.Level_Range;
begin

  -- By virtue of the fact that this procedure has been called means
  -- that the level ranges must be populated so set to a senseless
  -- null value guaranteed to be overwritten
  The_Range.Lower := Altitudes.Flight_Level_T'Last;
  The_Range.Upper := Altitudes.Flight_Level_T'First;

  for Idx in PIO_Data.Trajectory_Index_T range 1 .. Quantity loop
    --# assert Quantity = Quantity%;

    -- Must have a standard occupancy at the very least so check for that
    if MTCO_Types.Get_Standard_Occupancy (Segs_Group (Idx)).Exists then

      Temp_Range := Segment_FL_Occupancy (Segs_Group (Idx));

      The_Range.Lower :=
        Altitudes.Flight_Level_T'Min (The_Range.Lower,
                                       Temp_Range.Lower);

      The_Range.Upper :=
        Altitudes.Flight_Level_T'Max (The_Range.Upper,
                                       Temp_Range.Upper);

    end if;

  end loop;

  return The_Range;
end Segment_Group_FL_Occupancy;
```

# SPARK Training

- 57 people trained in SPARK
  - Mostly contractors and clients.
  - Diverse programming background.
  - All SPARK coders are also Z readers.
- Effective as SPARK coders immediately
- Picking up RTE proof takes longer.
  - About 2 months.
- How long to pick up formal correctness proofs?
  - No data, but I suspect longer again.

# SPARK Tools

- The SPARK toolset
  - Examiner.
  - Proof Simplifier.
  - Proof Checker.

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# Test Design

## 2.2.1.18 TPDeviationRequests

### Summary

Requests the required deviation trajectories.

This is a non-conditional schema.

### Partitions

There are two equivalence classes:

- 1 Flight is not radar supported, so no information.
- 2 Flight is radar supported.

The output condition in the first equivalence class is that there is no request. This can also occur when there are no deviation trajectories, so that input condition should be tested as well.

It is stated within the FPM process specification that the number of deviation requests will be either none, one or two ([4] section 13.2.13.2). We should test for each of these conditions separately (since 0 and 2 are boundary conditions).

### Test Conditions

TPDeviationRequests	1	2	3	4
<i>fpmData!?</i> = nil	●	○	○	○
<i>(the fpmData!?)</i> . <i>fpmDeviationTrajectories</i> = ∅		●	○	○
<i>deviationReqs</i> = ∅	●	●	○	○
<i>#deviationReqs</i> = 1	○	○	●	○
<i>#deviationReqs</i> = 2	○	○	○	●

# The Challenge of Test Design

*TPRemoveMultiplePIOs*

$\Delta TP$

*tpFlights!?* :  $\mathbb{P}FLIGHT$

*piosToRemove* :  $\mathbb{P}PIO$

$\exists$  *deletedDirectPIOs*, *deletedGroupedPIOs* :  $\mathbb{P}PIO$  |

*deletedDirectPIOs* = *flightPIO* (*tpFlights!?*)  $\cap$  *piosToRemove*

$\wedge$  *deletedGroupedPIOs* = *pioPIOGroup*<sup>~</sup> (*deletedDirectPIOs*) •

*flightPIO'* = *flightPIO*  $\triangleright$  *deletedDirectPIOs*

$\wedge$  *pioPIOGroup'* = *pioPIOGroup*  $\triangleright$  *deletedDirectPIOs*

$\wedge$  *pioGroupDisplayPIO'* = *deletedDirectPIOs*  $\triangleleft$  *pioGroupDisplayPIO*

$\wedge$  *pioState'* = (*deletedDirectPIOs*  $\cup$  *deletedGroupedPIOs*)  $\triangleleft$  *pioState*

*nominalVerticalProfiles'* =

**if** *fwpHookedFlight*  $\cap$  *tpFlights!?*  $\neq$  nil

**then** nil **else** *nominalVerticalProfiles*

How many potential tests for this fragment?

# The Challenge of Test Design

- If you just turn the handle there are **1134** conditions to test.
- But if you work at it hard enough you can cover the required subset in just **6** test scripts.
- Formal methods are not a substitute for initiative.

# Test reference models

- Algorithms are specified in pure mathematics.
  - Working out the expected answer for test cases is very difficult and error prone.
- We generate test cases as usual.
- We create a test reference implementation in Mathematica.
- We do back-to-back testing of iFACTS against the reference.
  - Diverse tools and implementers reduce the possibility of a common failure.

# Mathematica tools & training

- Small team – only 5 trained.
- Reference model has similar defect density to SPARK implementation.
- Limited conclusions to draw from such a small activity.

# Conclusions

- Formal methods are applicable to all phases of the lifecycle.
- Training engineers is not a barrier
  - It's a one-off cost
  - Our data shows that training is easy and cheap.
- Tool support is vital
  - The Achilles heel of formal methods
    - Except the SPARK Examiner!

# Altran Praxis Limited

20 Manvers Street  
Bath BA1 1PX  
United Kingdom

Website: [www.altran-praxis.com](http://www.altran-praxis.com)



# Tracing

- Completeness of coverage
  - e.g., testing all parts of a Z specification
- DOORS tool
  - Integrate Systems Engineering
- Link all specification components with test case(s) or argument for safety case
- Flag unlinked components
- Also visualization of schema structure



[www.integrate.biz/casestudies/BusinessGoalAlignment.aspx](http://www.integrate.biz/casestudies/BusinessGoalAlignment.aspx)

# More recent developments

- iFACTS in maintenance phase
- Traffic Load Prediction Device (TLPD)
- Forecast air traffic load up to 4 hours ahead
- Plan workloads for optimum traffic flows

[www.altran.co.uk/uksolutions/ecs/sectors/air-traffic-management.html](http://www.altran.co.uk/uksolutions/ecs/sectors/air-traffic-management.html)



# Reflection

*Oui, l'ouvre sort plus belle  
D'une forme au travail  
Rebelle,  
Vers, marbre, onyx, émail.*

[Yes, the work comes out more beautiful from a material that resists the *process*, verse, marble, onyx, or enamel.]

— Théophile Gautier (1811–1872) *L'Art*



# **Beware Panaceas!**



# PREPARATION Z

"The Shampoo That Shrinks Your Head To Maximize What Little Hair You Have Left!"



Before



After



**Beware  
Panaceas!**

Cf. Formal  
methods



**Caviat  
Emptor!**

# PREPARATION



**Caviat  
Emptor!**

# PREPARATION Z<sup>TM</sup>

**Caviat  
Emptor!**

Cf. Software

## CAUTIONS:

- **DO NOT USE ON SMALL HEADS**
- **BE PREPARED TO BECOME A MORON SINCE A SMALLER HEAD MEANS A SMALLER BRAIN WHICH MEANS A SMALLER I.Q.**
- **BE PREPARED FOR ADDITIONAL CLOTHING EXPENSES AS HAT & COLLAR SIZES WILL CHANGE.**
- **MEN SHOULD BE EXTREMELY CAREFUL IN USING PREPARATION Z AS ALL BODY PARTS ARE SUBJECT TO SHRINKAGE.**

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SOLD AS A NOVELTY ITEM ONLY!



# PREPARATION Z<sup>TM</sup>

**Caviat  
Emptor!**

Cf. Software

## CAUTIONS:

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# **The Industrial Use of Formal Methods: Experiences of an Optimist**

**Prof. Jonathan P. Bowen**

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